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Coherent light scattering in an Aharonov–Bohm geometry of quantum wells

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Abstract. This paper discusses the possibility of the transfer of electronic Aharonov–Bohm phase factors to light in double-quantum-well structures in magnetic fields. The open nature of semiconductor quantum wells permits a new type of coherent light scattering by means of certain interband transitions which preserve the positional indeterminacy of electronic states in Aharonov–Bohm superpositions. The modes of the coherently scattered light are determined by the magnetic flux per unit length between the wells. The calculation shows that the scattering mechanism can be quite efficient and may be used as an optical probe of ballistic electrons whose wavefunctions do not suffer a reduction while traversing the wells.

1. Introduction

Several years ago the Aharonov–Bohm effect was demonstrated in the conductivity of a double-quantum-well structure (Datta *et al* 1985). The open nature of such structures offers the possibility of a new type of light scattering from electrons which are in Aharonov–Bohm superpositions of Landau orbits centred in different wells. In this paper we demonstrate that if the final states of electrons after the scattering are still some Aharonov–Bohm superpositions of Landau orbits, then the scattered light is coherent and its modes are determined by the magnetic flux per unit length between the quantum wells.

Traditionally, the Aharonov–Bohm effect has been associated with isolated paths for electrons. What is meant by the phrase 'isolated paths' here is that the wavefunctions corresponding to different paths do not have any finite overlap. Indeed, in their original paper, Aharonov and Bohm (1959) presented a thought experiment in which electronic paths were completely isolated, yet they led to quantum interferences which illustrated in a striking fashion the special significance of potentials compared with forces in the quantum theory. However, such completely isolated paths are not necessary for the Aharonov–Bohm effect. There will be Aharonov–Bohm interferences as long as there are two alternative paths for an electron which is in a superposition of the states corresponding to these paths, and if these states accumulate different phases along their respective paths. There is, in principle, no restriction on the widths of the wavefunctions which appear in a superposition state, and there may or may not be a finite overlap between the two wavefunctions corresponding to the alternative paths.

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A semiconductor microstructure like the one used by Datta *et al* provides an example in which electronic paths are not isolated. In these structures a Landau state wavefunction centred in one well will have a tail in the neighbouring well for sufficiently narrow wells and for appropriate energies. Such a leakage of wavefunctions permits, under the right conditions, optical transitions of electrons from one well to another, thereby leading to the transfer of Aharonov–Bohm phases from electrons to optical fields.

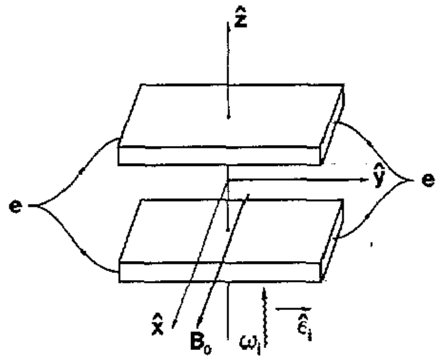

Figure 1. The double-quantum-well geometry for the Aharonov–Bohm effect. The slabs represent the quantum wells. The crystal in which these wells are embedded and the contacts through which electrons are injected and extracted are not shown.

If one chooses the Landau gauge for the geometry of figure 1

$$A_0 = -\hat{y}B_0 z \qquad B = \nabla \times A = B_0 \hat{x} \tag{1.1}$$

where B_0 is the magnetic field, an arbitrary Landau state may be designated as $|bnk_xk_y\rangle$, where b labels the band, $n = 0, 1, 2\ldots$ is the Landau index for the oscillator states, and $(k_x, k_y) = k$ is the electronic wavevector. To designate the quantum-well-centred Landau states, one can reduce the electronic wavevector as follows (Elçi and Depatie 1990):

$$k = K + \hat{y}wK_m \tag{1.2a}$$

where $w = \pm 1$ and

$$K_m = \frac{(z_0 + L/2)}{l_B^2} \qquad l_B = \sqrt{\frac{c\hbar}{eB_0}}. \tag{1.2b}$$

L is the width of a well, and $2z_0 + L$ is the separation between the centres of the wells. If $|K_y| < (L/2l_B^2)$, the quantum-well-centred states are associated with w and may be designated as $|bnwK\rangle$. $w = +1$ represents a Landau state which is centred in the well located at $z = z_0 + L/2$ (see figure 2). $w = -1$ represents a Landau state which is centred in the well located at $z = -(z_0 + L/2)$. We should emphasize that it is the centre of a cyclotron orbit which is confined to a well, not the entire orbit. Thus the presence of the quantum wells introduces an effective quantum number $w = \pm 1$.

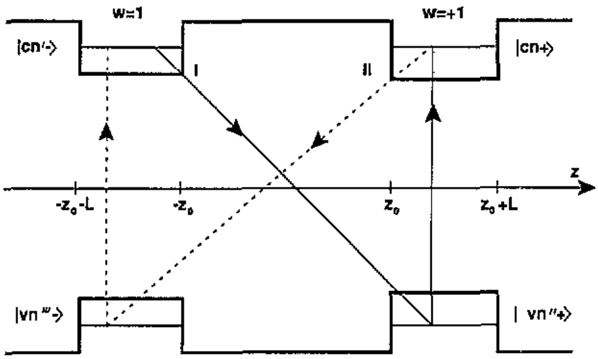

Figure 2. Interband transitions of the quantum-well-centred Landau states for coherent light scattering.

The Aharonov–Bohm superposition states correspond to superpositions of degenerate Landau states which have different w.

If the longitudinal ($\parallel \hat{z}$) voltage drop across the structure in figure 1 is zero or negligible, then a ballistic net current along the y-axis is obtained only if electrons are injected into the wells in Aharonov–Bohm superposition states (Elçi and Depatie 1990). Let us consider two conduction band Landau states $|cn+\rangle$ and $|cn'-\rangle$ which are degenerate in energy. For simplicity we suppressed the electronic wavevectors. An example for an Aharonov–Bohm superposition state is

$$|\psi^{\mathrm{AB}}(\theta)\rangle = \frac{1}{\sqrt{2}}\left(\mathrm{e}^{\mathrm{i}\theta}|cn'-\rangle + |cn+\rangle\right). \tag{1.3}$$

Thus the position of the electron is indeterminate with respect to the wells. If an electron in the state $|\psi^{\mathrm{AB}}\rangle$ is coupled to an incident coherent light beam, both components of the superposition state undergo transitions unless the electron suffers a wavefunction reduction. If such a reduction does not occur so that optical transitions preserve the positional indeterminacy, that is, if the final electronic state is still an Aharonov–Bohm superposition state, then the incident light is coherently scattered into certain modes which are determined by K_m. By coherent scattering we mean that if the incident field is a plane wave of a certain polarization, then the scattered field is also a plane wave of a definite polarization. In section 2 we show that for an incident beam propagating along the $\hat{z}$ direction, the coherently generated modes have the transverse wavevector components

$$\boldsymbol{q}_{s\perp} = \pm 2K_m\hat{\boldsymbol{y}}. \tag{1.4}$$

These components essentially determine the propagation directions of the scattered waves.

If the components of $|\psi^{\mathrm{AB}}\rangle$ undergo the following virtual interband transitions, the positional indeterminacy is preserved and the final state is an Aharonov–Bohm superposition state. Consider the sets of transitions illustrated in figure 2, where $|vn''+\rangle$ and $|vn'''-\rangle$ are two degenerate Landau states in a valence band. In the set of transitions labelled I, the first component of $|\psi^{\mathrm{AB}}\rangle$, namely $|cn'-\rangle$ is changed into $|vn''+\rangle$ by a virtual transition in which a photon of wavevector $\boldsymbol{q}_s$ is emitted; then a second virtual transition changes $|vn''+\rangle$ into $|cn+\rangle$ by absorption of an incident photon $\boldsymbol{q}_i$. Thus, these two virtual transitions create a counterclockwise circulation around the magnetic field and in the following section we show that the scattered photon has $\boldsymbol{q}_{s\perp} = -2K_m\hat{\boldsymbol{y}}$. In the set of transitions labelled II, the second component $|cn+\rangle$ of $|\psi^{\rm AB}\rangle$ is changed into $|vn'''-\rangle$ by the emission of a photon; then $|vn'''-\rangle$ is changed into $|cn'-\rangle$ by the absorption of an incident photon. This second set of virtual transitions creates a clockwise circulation around the magnetic field lines and the outgoing photon has $\boldsymbol{q}_{s\perp} = 2K_m\hat{\boldsymbol{y}}$. When an electron undergoes the four virtual transitions shown in figure 2 through its component states in (1.3), the final state is again an Aharonov–Bohm superposition state which can be written as $\exp(\mathrm{i}\theta')|\psi^{\rm AB}(\theta'')\rangle$. Note that the non-isolation of the quantum wells is essential for inducing the overall transition $|\psi^{\rm AB}(\theta)\rangle \to \mathrm{e}^{\mathrm{i}\theta'}|\psi^{\rm AB}(\theta'')\rangle$. If the wells are completely isolated, transitions like $|cn'-\rangle \to |vn''+\rangle$ cannot take place. Note also that although the electron undergoes four interband transitions, the overall scattering process is second-order, not fourth-order. The whole process described above may be visualized as the coherent superposition of two distinct Compton scatterings.

One can understand the imparting of the transverse momenta of (1.4) to scattered photons qualitatively from the following classical picture. The onset of the charge circulations around the magnetic field lines associated with the virtual transitions develops EMF forces, mostly along the y-axis. When an electric field along the y-axis acts upon a Landau state in the geometry of figure 1, it induces a net probability current along the z-axis (Elçi and Depatie 1990). Thus, incident photons cause the electronic charge in the superposition state to act effectively as an oscillating dipole which is nearly parallel to the z-axis and which emits photons propagating along the y-axis. Note that if an electron is described by a density matrix $\boldsymbol{\rho}^{\rm AB}$, then equation (1.3) implies that $\boldsymbol{\rho}^{\rm AB}$ has off-diagonal matrix elements with respect to the well index w. This is analogous to the dipole moment of an atom which can couple to a radiation field (Sargent *et al* 1974). The off-diagonal elements of $\boldsymbol{\rho}^{\rm AB}$ may also be interpreted as a dipole moment. It should be noted, however, that the states involved in the atomic polarization generally have different energies. In contrast, in $\boldsymbol{\rho}^{\rm AB}$ the states have the same energy.

If the Aharonov–Bohm superposition state suffers a reduction, or the electron is in a state which is centred in one well to begin with, then the calculation of the following section reduces to the ordinary light scattering, in that one part of the incident coherent beam is reflected back and the remaining part propagates forward. On the other hand, if the electron is in an Aharonov–Bohm superposition state initially, it can undergo the transitions I and II together without suffering any reduction, as described in the preceding paragraph. After the four transitions shown in figure 2, one has essentially $|\psi^{\rm AB}(\theta)\rangle \to \exp(\mathrm{i}\theta')|\psi^{\rm AB}(\theta'')\rangle$ and the electron remains in an Aharonov–Bohm superposition state. Its position with respect to the wells is still indeterminate in the final state.

This positional indeterminacy of the final electronic states forces the states of the scattered field to be coherent. The overall transition $|\psi^{\rm AB}(\theta)\rangle \to \exp(\mathrm{i}\theta')|\psi^{\rm AB}(\theta'')\rangle$ cannot take place if the states of the scattered field are number states. This can be seen from the following thought experiment (Elçi 1989). Let us imagine that two light detectors are placed on the left and the right of the microstructure, on the y-axis. Let us also imagine that the ballistic current is sufficiently reduced so that at any moment there is at most one ballistic electron in the structure. One can therefore, in principle, associate the scattered photons detected within a time interval with a given ballistic electron. If the scattered light is in a definite number state, one can deduce the position of the electron. For example, if just one photon is detected by the left detector, while the right detector indicates zero photons, then the electron must have been initially in the well labelled by $w = -1$, and it must have gone to the well labelled by $w = +1$, since the transverse momentum imparted to the photon is $-2K_m\hat{y}$ and the electron must have undergone the set of transitions I without undergoing the transitions II. In other words, the original Aharonov–Bohm superposition wavefunction must have collapsed into a single quantum-well-centred Landau wavefunction. Similarly, a photon at the right detector implies that the final position of the electron is in the well $w = -1$. It follows that the state $\exp(i\theta')|\psi^{AB}(\theta'')\rangle$ cannot be associated with definite number states for the scattered light. Rather, the observed numbers of the scattered photons to the left and the right must be indefinite, but the phases associated with the scattering modes must be definite, in order to prevent destructive interferences between the two plane waves originating from the same electron, one going to the left, the other to the right. Thus the light scattering from electrons which remain in Aharonov–Bohm superposition states must be coherent. The calculation presented in the following section and in the appendices confirms this qualitative picture.

In the transitions of figure 2, the intermediate valence band states $|vn''+\rangle$ and $|vn'''-\rangle$ are centred in the quantum wells. There are also valence band Landau states which are not centred in any of the wells and one might wonder whether they would contribute to the coherent scattering. In fact, they do not. The theory indicates that as long as $|cn+\rangle$ and $|cn'-\rangle$ are quantum well centred states, the intermediate states must also be centred in the quantum wells in order to scatter the incident photons coherently. We should also add that the intermediate Landau states need not be in the valence band, but could be in a conduction band c' which is higher than c. The order of absorptions and emissions are interchanged in this case, but the essential physical picture remains unchanged.

Note also that with the completion of the four virtual transitions the original superposition state is nearly recovered except for a relative phase change. Since the relative phase can be related to the current densities at the boundaries, the effect of the above radiative transitions is essentially to induce in the environment an adiabatic transformation of the kind in Berry's theory (Berry 1984).

In section 2, we describe the Hamiltonian and its representation in terms of the Landau states in the presence of the quantum well potential and a pair of applied DC electric fields. This Hamiltonian is used to calculate the density matrix operator to second order in the optical field amplitudes, assuming that the coupling between the incident field and the crystal structure is turned on at $t = 0$. The second-order density matrix for large t is used to determine the expectation value of the coherently generated scattering modes from electrons which are in Aharonov–Bohm superposition states. The details of the calculation are summarized in appendices A and B. We also discuss the mode equations and the efficiency of the scattering mechanism for the geometry of figure 1, where the incident field propagates parallel to the z-axis and the Hall field along the z-axis is negligible. The magnetic flux induced coherent scattering is enhanced under certain resonance conditions which relate the incident field frequency to the magnetic field, the spatial separation of the wells, and to the energy separation between the bands involved in the virtual transitions. The theory indicates that the proposed scattering mechanism can be quite efficient and can be observed experimentally in structures similar to the one in the original Aharonov–Bohm experiment (Datta *et al* 1985). The analysis of section 2 is carried out for a symmetric double-quantum-well structure, which simplifies the calculation and its results. The proposed coherent light scattering is, however, independent of such symmetry. By means of a simple substitution, the formulas are readily modified for the case in which the widths of the two wells differ. This is discussed in section 3.

In addition to its novelty, the proposed magnetic-flux-induced coherent light scattering might have useful applications in steering of laser beams magnetically, or to semiconductor lasers for selecting the lasing modes. We believe, however, that its most interesting application would be as an optical probe of the reduction or non-reduction of the electronic superposition states. The amplitudes of the coherently generated scattering modes measure directly the number of ballistic electrons whose positions are indeterminate and whose wavefunctions have not suffered any reduction as a result of the light scattering. The efficiency of the scattering process suggests that such an optical probe would be more sensitive than a conductivity experiment.

2. Coherent light scattering

In this section we describe the Hamiltonian used in the second-order scattering calculation and give the results for the coherently scattered field. The details of the calculation are summarized in appendices A and B.

Since our discussion is concerned with ballistic electrons, we may write the one-electron Hamiltonian in terms of the projection operators for the electronic states:

$$H = H_0 + H_1 \tag{2.1a}$$

$$H_0 = \sum_\alpha E_\alpha |\alpha\rangle\langle\alpha| + \sum_\mu \hbar\omega_\mu b_\mu^\dagger b_\mu \tag{2.1b}$$

$$H_1 = \sum_{\alpha\alpha'\mu} \left[\hbar g^\mu_{\alpha\alpha'} |\alpha\rangle\langle\alpha'| b_\mu + \text{HC}\right] \tag{2.1c}$$

$$g^\mu_{\alpha\alpha'} = -\frac{\mathrm{i}e}{m}\left(\frac{2\pi\hbar}{\omega_\mu V_{ol}\epsilon_\infty}\right)^{1/2} \int \mathrm{d}\boldsymbol{x}\, \psi^*_\alpha(\boldsymbol{x}) \exp(\mathrm{i}\boldsymbol{q}_\mu \cdot \boldsymbol{x}) \hat{\epsilon}_\mu \cdot \frac{\partial}{\partial \boldsymbol{x}} \psi_{\alpha'}(\boldsymbol{x}). \tag{2.1d}$$

In (2.1*b*), the $|\alpha\rangle$ represent the Landau states in the quantum-well structure. We assume that they are calculated in the effective mass approximation near the centre of the Brillouin zone (BZ). We also assume that the bands of interest are isotropic, which permits us to use the results of Elçi and Depatie (1990). Let $\mathcal{E}_y$ and $\mathcal{E}_z$ be the DC electric fields in the structure. A Landau state wavefunction modified by the potential of the wells, as well as by $\mathcal{E}_y$ and $\mathcal{E}_z$, has the form (Elçi and Depatie 1990)

$$\psi_\alpha(\boldsymbol{x}) = \psi_{bnw\boldsymbol{K}}(\boldsymbol{x}) = \frac{\exp[\mathrm{i}(\boldsymbol{K} + wK_m\hat{\boldsymbol{y}})\cdot\boldsymbol{x}]}{\sqrt{\mathcal{L}_x\mathcal{L}_y}} \phi_{bnwK_y}(z)\psi^{\mathrm{B}}_{b0}(\boldsymbol{x}) \tag{2.3}$$

where $\psi^{\mathrm{B}}_{b0}(\boldsymbol{x})$ is the Bloch function of the band b at the centre of the BZ. $\mathcal{L}_x$ and $\mathcal{L}_y$ are the sample dimensions in the $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ directions. $\phi_{bnwK_y}(z)$ is approximately given by

$$\phi_{bnwK_y}(z) = (1 + \mathrm{i}\sigma_b\kappa_b z_{bw})u_n(z_{bw}) + \sum_{n'} \beta^b_{nn'} u_{n'}(z_{bw}) \tag{2.4}$$

where

$$z_{bw} = z - \left(wK_m + K_y\right) l_B^2 + \frac{e\mathcal{E}_z \sigma_b}{m_b^* \omega_{bc}^2} \tag{2.5a}$$

$$\kappa_b = \frac{e\mathcal{E}_y}{\hbar\omega_{bc}} \qquad w_{bc} = \frac{eB_0}{m_b^* c}. \tag{2.5b}$$

Here m_b^* is the effective mass of the band b. σ_b represents the sign of the curvature of the band at the centre of the BZ:

$$\sigma_b = \mathrm{sign}\left((\mathrm{d}^2 E_b/\mathrm{d}k^2)_{k=0}\right). \tag{2.6}$$

Here $u_n(z)$ is the harmonic oscillator wavefunction:

$$u_n(z) = \left(l_B\sqrt{\pi}2^n n!\right)^{-1/2} \exp(-z^2/2l_B^2) H_n\left(\frac{z}{l_B}\right). \tag{2.7}$$

Here H_n is the Hermite polynomial. $\beta^b_{nn'}$ in (2.4) arises from the quantum well potential. For sufficiently shallow wells and for the Landau states centred in these wells, $\beta^b_{nn'}$ is given approximately by (Elçi and Depatie 1990)

$$\beta^b_{nn'} = (1-\delta_{nn'})\delta W^b_{nn'}[\hbar\omega_c(n-n')]^{-1} \tag{2.8a}$$

$$\begin{aligned}\delta W^b_{nn'} = \sigma_b W_{b0}\left[\int_{L/2}^{\infty} \mathrm{d}z\, u_n(z)u_{n'}(z) + \int_{-\infty}^{-L/2} \mathrm{d}z\, u_n(z)u_{n'}(z)\right] \\ + \sigma_b(-1)^{n+n'+1} W_{b0} L u_n(2l_B^2 K_m) u_{n'}(2l_B^2 K_m)\end{aligned} \tag{2.8b}$$

where W_{b0} is the height of the potential well for the band b. ϕ_{bnwK_y} represents a quantum-well-centred Landau state if $|K_y| < (L/2l_B^2)$. If $|K_y| > (L/2l_B^2)$, the Landau state is centred outside the wells. In this case we assume that the reduction of k_y defined in (1.2a) is carried out by choosing the sign of w such that the resulting $|K_y|$ is minimum. Then the label w in ϕ_{bnwK_y} represents the nearest well to the centre of the Landau orbit.

Returning to equations (2.1), b_μ and $b^\dagger_\mu$ are the annihilation and creation operators for the photon mode μ. $\hat{\epsilon}_\mu$ and $\vec{q}_\mu$ are the polarization and the propagation vectors of this mode. V_{ol} is the quantization volume. To simplify the problem, we assigned a single index of refraction $\sqrt{\epsilon_\infty}$ for the entire structure. H_1 represents the $\boldsymbol{p}\cdot\boldsymbol{A}$ coupling. In the representation of ψ_{bnwK}, the coupling coefficient g is given by [see appendix A]

$$\begin{aligned}g^\mu_{bnw;b'n'w'}(\boldsymbol{K},\boldsymbol{K}') \simeq \frac{e}{m}\left(\frac{2\pi}{\epsilon_\infty V_{ol}\hbar\omega_\mu}\right)^{1/2} \hat{\epsilon}_\mu\cdot\boldsymbol{P}_{bb'} \\ \times\, \delta_{\boldsymbol{K}+wK_m\hat{\boldsymbol{y}};\boldsymbol{K}'+w'K_m\hat{\boldsymbol{y}}+q_{\mu\perp}} M_{bnw;b'n'w'}(K_y, K'_y|q_{\mu z})\end{aligned} \tag{2.9a}$$

where $\boldsymbol{P}_{bb'}$ is the interband momentum matrix element and

$$M_{bnw;b'n'w'}(K_y, K'_y|q_z) = \int \mathrm{d}R_z\, \exp(\mathrm{i}q_z R_z)\phi^*_{bnwK_y}(R_z)\phi_{b'n'w'K'_y}(R_z). \tag{2.9b}$$

We will be concerned with those magnetic fields and quantum well structures such that $(q_{\mu z})^{-1} > l_B, L, z_0$ for optical or longer wavelengths. One may therefore set $q_{\mu z}$ to zero in (2.9*b*). Furthermore, if one takes just the leading terms $u_n(z_{bw})$ in the expression for ϕ_{bnwK_y} in (2.4), one finds that

$$M_{bnw;b'n'w'}(K_y, K_y'|0) \simeq \exp(-\Lambda^2/4) D_{nn'}(\Lambda) \tag{2.10a}$$

where

$$\Lambda = (w - w')l_B K_m + l_B(K_y - K_y') - \frac{e\mathcal{E}_z}{l_B}\left(\frac{\sigma_b}{m_b^* \omega_{bc}^2} - \frac{\sigma_{b'}}{m_{b'}^* \omega_{b'c}^2}\right) \tag{2.10b}$$

and

$$D_{nn'}(x) = \sqrt{\frac{n!n'!}{2^{n+n'}}} \sum_\lambda \frac{(-1)^{n-\lambda} 2^\lambda x^{n+n'-2\lambda}}{(n-\lambda)!(n'-\lambda)!\lambda!}. \tag{2.10c}$$

In (2.10*c*), the factorial of a negative integer in the denominator makes the summand zero. For order of magnitude estimates, the approximation (2.10*a*) is quite sufficient.

Finally, the wavefunction ψ_{bnwK} is associated with the eigenenergy

$$E_{bnw}(\boldsymbol{K}) = E_{b0} + c\hbar(wK_m + K_y)\left(\frac{\mathcal{E}_z}{B_0}\right) + \sigma_b\left[\hbar\omega_{bc}(n + \tfrac{1}{2}) + \frac{\hbar^2 K_x^2}{2m_b^*} + \delta W_{bn}\right]. \tag{2.11}$$

Here E_{b0} is a constant which represents the separation of the band edge of b from the reference zero energy. δW_{bn} is the correction induced by the quantum well potential in the Landau state energy. It may depend on $(K_y + wK_m)$. Under the approximation when equations (2.8) hold, δW_{bn} for the quantum-well-centred states is given by (Elçi and Depatie 1990)

$$\delta W_{bn} \approx \delta W_{nn}^b. \tag{2.12}$$

When the Landau state is not centred in the wells, $\delta W_{bn} \approx W_{b0}$.

Two remarks are in order before we consider the scattered field. Firstly, the electron–photon coupling represented by H_1 is a simplification. The full coupling Hamiltonian is given by

$$H_1' = \frac{1}{m}\left(\boldsymbol{p} + \frac{e}{c}\boldsymbol{A}_0\right) \cdot \boldsymbol{A} + \frac{e^2}{2mc^2}\boldsymbol{A}^2 \tag{2.13}$$

where $\boldsymbol{A}$ and $\boldsymbol{A}_0$ are the vector potentials for the radiation and magnetic fields, respectively. Since our discussion is concerned with radiative interband transitions, we omitted in (2.1) the couplings which are of the form $\boldsymbol{A}^2$ and $\boldsymbol{A}_0 \cdot \boldsymbol{A}$. Secondly, ϕ_{bnwK_y} in (2.4) and $E_{bnw}(\boldsymbol{K})$ in (2.11) were derived by Elçi and Depatie (1990) under the assumptions that $\kappa_b l_B < 1$, $|\beta_{nn'}^b| < 1$, and $z_0 + L/2 > l_B$. The last assumption was made to simplify certain integrals. In the experiment that we propose in the present paper, we actually consider cases in which $z_0 + L/2 < l_B$. This has some effect on the actual numerical values of $\beta_{nn'}^b$ and δW_{bn}. We expect, however, these values to be not too different from (2.8) and (2.12) for the purpose of order-of-magnitude estimates.

The Hamiltonian given in (2.1) may be used to calculate the density matrix operator of the electrons plus the field to second order in the incident field amplitude as time becomes indefinitely large. This density matrix may then be used to calculate the expectation value of the vector potential field, in particular the contribution of the electrons which remain in Aharonov–Bohm superposition states to this expectation value. We assume that a coherent field mode is incident on the structure at $t = 0$ and that the overall density matrix is separable at $t = 0$:

$$\rho(0) = \rho^{\mathrm{e}}(0) \otimes \rho^{\mathrm{F}}(0) \tag{2.14}$$

where $\rho^{\mathrm{e}}(0)$ is the electronic part and $\rho^{\mathrm{F}}(0)$ is the field part. Let

$$\langle b_\mu \rangle = \mathrm{Tr}\,\left(b_\mu \rho^{\mathrm{F}}(0)\right) = \delta_{\mu,i} \exp(\mathrm{i}\phi_i)\sqrt{\bar{n}_i} \tag{2.15}$$

where ϕ_i is the phase of the incident coherent mode and $\bar{n}_i$ is the average number of quanta in it. Also let

$$\mathrm{Tr}\,(|\alpha\rangle\langle\alpha'|\rho^{\mathrm{e}}(0)) = \rho^{\mathrm{e}}_{\alpha'\alpha}(0). \tag{2.16}$$

As stated in the introduction, $\rho^{\mathrm{e}}_{\alpha\alpha'}$, has off-diagonal matrix elements between degenerate quantum-well-centred states with $w = -w'$ for the electrons which are in Aharonov–Bohm superposition states. We consider only the contribution of these off-diagonal matrix elements in the scattering:

$$\rho^{\mathrm{e}}_{\alpha\alpha'} \rightarrow \rho^{\mathrm{AB}}_{bnw;bn'-w}(\boldsymbol{K}, \boldsymbol{K}') \qquad |K_y|, |K'_y| < (L/2l_B^2) \qquad E_{bnw}(\boldsymbol{K}) = E_{bn'-w}(\boldsymbol{K}'). \tag{2.17}$$

This scattering calculation is summarized in appendix B. One finds that the coherently generated scattering modes arising from ρ^{AB} are the solutions of any one of the following four sets of the equations:

$$\begin{aligned} \boldsymbol{q}_{s_1\perp} &= \boldsymbol{K} - \boldsymbol{K}' + 2wK_m\hat{\boldsymbol{y}} + \boldsymbol{q}_{i\perp} \qquad \hat{\epsilon}_{s_1}\cdot\boldsymbol{q}_{s_1} = 0 \\ \hbar\omega_{s_1} &= E_{bn'-w}(\boldsymbol{K}') - E_{b''n''w''}(\boldsymbol{K}' - (w+w'')K_m\hat{\boldsymbol{y}} - \boldsymbol{q}_{i\perp}) \\ \hat{\boldsymbol{z}}\cdot\boldsymbol{q}_{s_1} &= \pm\left[\frac{\epsilon_\infty\omega_{s_1}^2}{c^2} - (2wK_m\hat{\boldsymbol{y}} + \boldsymbol{q}_{i\perp} + \boldsymbol{K} - \boldsymbol{K}')^2\right]^{1/2} \end{aligned} \tag{2.18a}$$

$$\begin{aligned} \boldsymbol{q}_{s_2\perp} &= \boldsymbol{K} - \boldsymbol{K}' + 2wK_m\hat{\boldsymbol{y}} - \boldsymbol{q}_{i\perp} \qquad \hat{\epsilon}_{s_2}\cdot\boldsymbol{q}_{s_2} = 0 \\ \hbar\omega_{s_2} &= E_{bn'-w}(\boldsymbol{K}') - E_{b''n''w''}(\boldsymbol{K}' - (w+w'')K_m\hat{\boldsymbol{y}} + \boldsymbol{q}_{i\perp}) \\ \hat{\boldsymbol{z}}\cdot\boldsymbol{q}_{s_2} &= \pm\left[\frac{\epsilon_\infty\omega_{s_2}^2}{c^2} - (2wK_m\hat{\boldsymbol{y}} + \boldsymbol{K} - \boldsymbol{K}' - \boldsymbol{q}_{i\perp})^2\right]^{1/2} \end{aligned} \tag{2.18b}$$

$$\begin{aligned} \boldsymbol{q}_{s_3\perp} &= \boldsymbol{K} - \boldsymbol{K}' + 2wK_m\hat{\boldsymbol{y}} + \boldsymbol{q}_{i\perp}, \quad \hat{\epsilon}_{s_3}\cdot\boldsymbol{q}_{s_3} = 0 \\ \hbar\omega_{s_3} &= E_{b''n''w''}(\boldsymbol{K} + (w-w'')K_m\hat{\boldsymbol{y}} + \boldsymbol{q}_{i\perp}) - E_{bnw}(\boldsymbol{K}) \\ \hat{\boldsymbol{z}}\cdot\boldsymbol{q}_{s_3} &= \pm\left[\frac{\epsilon_\infty\omega_{s_3}^2}{c^2} - (2wK_m\hat{\boldsymbol{y}} + \boldsymbol{K} - \boldsymbol{K}' + \boldsymbol{q}_{i\perp})^2\right]^{1/2} \end{aligned} \tag{2.18c}$$

$$\begin{aligned} \boldsymbol{q}_{s_4\perp} &= \boldsymbol{K} - \boldsymbol{K}' + 2wK_m\hat{\boldsymbol{y}} - \boldsymbol{q}_{i\perp} \qquad \hat{\epsilon}_{s_4}\cdot\boldsymbol{q}_{s_4} = 0 \\ \hbar\omega_{s_4} &= E_{b''n''w''}(\boldsymbol{K} + (w-w'')K_m\hat{\boldsymbol{y}} - \boldsymbol{q}_{i\perp}) - E_{bnw}(\boldsymbol{K}) \\ \hat{\boldsymbol{z}}\cdot\boldsymbol{q}_{s_4} &= \pm\left[\frac{\epsilon_\infty\omega_{s_4}^2}{c^2} - (2wK_m\hat{\boldsymbol{y}} + \boldsymbol{K} - \boldsymbol{K}' - \boldsymbol{q}_{i\perp})^2\right]^{1/2} \end{aligned} \tag{2.18d}$$

where $s_1, \ldots, s_4$ denote the modes. Any solutions of these equations are coherently excited by the ballistic electrons which are in Aharonov–Bohm superposition states. The second-order, coherently generated field is given by

$$
\begin{aligned}
\langle A(x,t)\rangle^{(2)}_{\rm coh} = {} & \frac{\pi e^2 N}{m^2 c}\left(\frac{2\pi\hbar\bar{n}_i}{\epsilon_\infty V_{ol}\omega_i}\right)^{1/2} \sum_{bb''nn'n''ww''KK'} \mathrm{Re}\Bigg\{ \mathrm{i}\rho^{\rm AB}_{bnw;bn'-w}(K,K') \\
& \times \Bigg[\sum_{(s_1)} \frac{\hat{\epsilon}_{s_1}\exp[\mathrm{i}(q_{s_1}\cdot x - \omega_{s_1}t + \phi_i)]}{|\hat{z}\cdot q_{s_1}|(\hbar\omega_{s_1} - \hbar\omega_i + \mathrm{i}\delta)}\hat{\epsilon}_{s_1}\cdot P^*_{bb''}\hat{\epsilon}_i\cdot P_{bb''} \\
& \times M^*_{bnw;b''n''w''}(K_y, K'_y - (w+w'')K_m - q_{iy}|\hat{z}\cdot q_{s_1}) \\
& \times M_{bn'-w;b''n''w''}(K'_y, K'_y - (w+w'')K_m - q_{iy}|\hat{z}\cdot q_i) \\
& + \sum_{(s_2)} \frac{\hat{\epsilon}_{s_2}\exp[\mathrm{i}(q_{s_2}\cdot x - \omega_{s_2}t - \phi_i)]}{|\hat{z}\cdot q_{s_2}|(\hbar\omega_{s_2} + \hbar\omega_i + \mathrm{i}\delta)}\hat{\epsilon}_{s_2}\cdot P^*_{bb''}\hat{\epsilon}_i\cdot P^*_{b''b} \\
& \times M^*_{bnw;b''n''w''}(K_y, K'_y - (w+w'')K_m + q_{iy}|\hat{z}\cdot q_{s_2}) \\
& \times M^*_{b''n''w'';bn'-w}(K'_y - (w+w'')K_m + q_{iy}, K'_y|\hat{z}\cdot q_i) \\
& - \sum_{(s_3)} \frac{\hat{\epsilon}_{s_3}\exp[\mathrm{i}(q_{s_3}\cdot x - \omega_{s_3}t + \phi_i)]}{|\hat{z}\cdot q_{s_3}|(\hbar\omega_{s_3} - \hbar\omega_i + \mathrm{i}\delta)}\hat{\epsilon}_{s_3}\cdot P^*_{b''b}\hat{\epsilon}_i\cdot P_{b''b} \\
& \times M^*_{b''n''w'';bn'-w}(K_y + (w-w'')K_m + q_{iy}, K'_y|\hat{z}\cdot q_{s_3}) \\
& \times M_{b''n''w'';bnw}(K_y + (w-w'')K_m + q_{iy}, K_y|\hat{z}\cdot q_i) \\
& - \sum_{(s_4)} \frac{\hat{\epsilon}_{s_4}\exp[i(q_{s_4}\cdot x - \omega_{s_4}t - \phi_i)]}{|\hat{z}\cdot q_{s_4}|(\hbar\omega_{s_4} + \hbar\omega_i + \mathrm{i}\delta)}\hat{\epsilon}_{s_4}\cdot P^*_{b''b}\hat{\epsilon}_i\cdot P^*_{bb''} \\
& \times M^*_{b''n''w'';bn'-w}(K_y + (w-w'')K_m - q_{iy}, K'_y|\hat{z}\cdot q_{s_4}) \\
& \times M^*_{bnw;b''n''w''}(K_y, K_y + (w-w'')K_m - q_{iy}|\hat{z}\cdot q_i)\Bigg]\Bigg\}
\end{aligned} \tag{2.19}
$$

where $N = \mathcal{N}/(\mathcal{L}_x\mathcal{L}_y)$ is the density of the ballistic electrons per unit area. It should be kept in mind that in (2.19), the sums over $s_1, \ldots, s_4$ are not independent sums but depend on the indices w, w'', b'', etc. We indicated this by putting parenthesis around s_i under the summation signs.

When the incident light propagates in the $\hat{z}$ direction, $q_{i\perp} = 0$ and the solutions s_1 and s_2 are indistinguishable. The same is true for s_3 and s_4. The modes pairwise merge together. Let $s_1, s_2 \to s$ and $s_3, s_4 \to s'$. The mode equations (2.18) may be simplified further by assuming that $K_x = K_{x'} = 0$ and $|K_y - K'_y| \ll 2K_m$. The first assumption means that the particle current is along the y-axis. The second means that the Landau states are centred near the midpoints of the wells. One then has

$$
\begin{aligned}
& q_{s\perp} = 2wK_m\hat{y} \qquad \hat{\epsilon}_s\cdot q_s = 0 \\
& \hbar\omega_s = E_{bn'-w}(K) - E_{b''n''w''}(K' - (w+w'')K_m\hat{y}) \\
& \hat{z}\cdot q_s = \pm\left[\frac{\epsilon_\infty\omega_s^2}{c^2} - 4K_m^2\right]^{1/2}
\end{aligned} \tag{2.20a}
$$

and

$$\begin{aligned} &\boldsymbol{q}_{s'\perp} = 2wK_m\hat{\boldsymbol{y}} \qquad \hat{\epsilon}_{s'}\cdot\boldsymbol{q}_{s'} = 0 \\ &\hbar\omega_{s'} = E_{b''n''w''}(\boldsymbol{K} + (w-w'')K_m\hat{\boldsymbol{y}}) - E_{bnw}(\boldsymbol{K}) \\ &\hat{\boldsymbol{z}}\cdot\boldsymbol{q}_{s'} = \pm\left[\frac{\epsilon_\infty\omega_{s'}^2}{c^2} - 4K_m^2\right]^{1/2}. \end{aligned} \tag{2.20b}$$

The corresponding expression for the scattered field can also be simplified. In (2.19), the first and third group of terms have resonance denominators. If the incident light frequency is adjusted to be near either ω_s or $\omega_{s'}$, these terms will be much larger than the second and fourth group of terms. We may therefore keep only those terms with resonance denominators and obtain

$$\begin{aligned} \langle \boldsymbol{A}(\boldsymbol{x},t)\rangle^{(2)}_{\mathrm{coh}} \simeq \frac{\pi e^2 N}{m^2 c}\left(\frac{2\pi\hbar\bar{n}_i}{\epsilon_\infty V_{ol}\omega_i}\right)^{1/2} \sum_{bb''nn'n''ww''\boldsymbol{KK}'} \mathrm{Re}\Big\{ \mathrm{i}\rho^{\mathrm{AB}}_{bnw;bn'-w}(\boldsymbol{K},\boldsymbol{K}') \\ \times\Bigg[\sum_{(s)} \frac{\hat{\epsilon}_s \exp[\mathrm{i}(\boldsymbol{q}_s\cdot\boldsymbol{x} - \omega_s t + \phi_i)]}{|\hat{\boldsymbol{z}}\cdot\boldsymbol{q}_s|(\hbar\omega_s - \hbar\omega_i + \mathrm{i}\delta)} \hat{\epsilon}_s\cdot\boldsymbol{P}^*_{bb''}\hat{\epsilon}_i\cdot\boldsymbol{P}_{bb''} \\ \times M^*_{bnw;b''n''w''}(K_y, K'_y - (w+w'')K_m|\hat{\boldsymbol{z}}\cdot\boldsymbol{q}_s) \\ \times M_{bn'-w;b''n''w''}(K'_y, K'_y - (w+w'')K_m|\hat{\boldsymbol{z}}\cdot\boldsymbol{q}_i) \\ - \sum_{(s')} \frac{\hat{\epsilon}_{s'} \exp[\mathrm{i}(\boldsymbol{q}_{s'}\cdot\boldsymbol{x} - \omega_{s'} t - \phi_i)]}{|\hat{\boldsymbol{z}}\cdot\boldsymbol{q}_{s'}|(\hbar\omega_{s'} - \hbar\omega_i + \mathrm{i}\delta)} \hat{\epsilon}_{s'}\cdot\boldsymbol{P}^*_{b''b}\hat{\epsilon}_i\cdot\boldsymbol{P}_{b''b} \\ \times M^*_{b''n''w'';bn'-w}(K_y + (w-w'')K_m, K'_y|\hat{\boldsymbol{z}}\cdot\boldsymbol{q}_{s'}) \\ \times M_{b''n''w'';bnw}(K_y + (w-w'')K_m, K_y|\hat{\boldsymbol{z}}\cdot\boldsymbol{q}_i)\Bigg]\Big\}. \end{aligned} \tag{2.21}$$

Note that two types of polarizations may be associated with (2.20). One has the polarization vector in the transverse plane, parallel to the magnetic field:

$$\hat{\epsilon} = \hat{\boldsymbol{x}}. \tag{2.22a}$$

The other has polarization vectors in the yz plane, perpendicular to the magnetic field:

$$\hat{\epsilon}_\mu = -[\mathrm{sign}(\hat{\boldsymbol{z}}\cdot\boldsymbol{q}_\mu)]\hat{\boldsymbol{y}}\cos\theta_\mu + [\mathrm{sign}(\hat{\boldsymbol{y}}\cdot\boldsymbol{q}_\mu)]\hat{\boldsymbol{z}}\sin\theta_\mu \tag{2.22b}$$

where

$$\theta_\mu = \tan^{-1}\left(\frac{2cK_m}{\sqrt{\epsilon_\infty\omega_\mu^2 - 4c^2K_m^2}}\right). \tag{2.22c}$$

We now discuss the implications of (2.21) for the geometry of figure 1. To simplify the following discussion, we assume that the longitudinal DC field $\mathcal{E}_z$ is negligible. Of course, many of the electrons injected into the structure do not have ballistic motions but scatter and collect near the well edges, which may set up a Hall field contributing to $\mathcal{E}_z$. For the following discussion we assume that if necessary, an external DC electric field along the z-axis is imposed on the system to compensate for the internal Hall field. It should be noted that the application of a DC voltage across the boundaries of the sample need not cancel the internal Hall field entirely, since the internal field may have a spatial structure inside the sample under steady state conditions. For the following discussion, we assume that this internal field variations produce only small, negligible perturbations in Landau state energies, so that contributions of $\mathcal{E}_z$ in (2.4) and (2.11) are negligible. This assumption has no effect on the coherent scattering mechanism but simplifies the algebra. We will also concern ourselves with the electronic states which produce particle currents only along the y-axis and are centred near the midpoints of the wells. We set $|\boldsymbol{K}| = |\boldsymbol{K}'| \approx 0$ and suppress $\boldsymbol{K}$ and $\boldsymbol{K}'$ in the matrix elements in (2.21), as well as in the mode equations. We also assume that δW_{bn} is given by (2.12). Under these assumptions (2.17) requires that $n = n'$ in $\rho^{\mathrm{AB}}_{bnw;bn'-w}$. We define

$$\bar{\rho}^{\mathrm{AB}}_{bnw;bn-w} = \sum_{KK'} \rho^{\mathrm{AB}}_{bnw;bn-w}(\boldsymbol{K}, \boldsymbol{K}'). \tag{2.23}$$

Let b and b'' be the conduction and valence bands, respectively, as in figure 2. In this case only the s-modes of equation (2.20a) are generated coherently and we need to take into account only the first group of terms in (2.21). Let $\sigma_c = +1$ and $\sigma_v = -1$. From (2.20a), the frequency of the coherently generated modes is given by

$$\hbar\omega_s = E_G + \hbar\omega_{cc}(n + \tfrac{1}{2}) + \hbar\omega_{vc}(n'' + \tfrac{1}{2}) + \delta W_{cn} + \delta W_{vn''} \tag{2.24}$$

where E_G is the band gap between the conduction and valence bands. If we let $w = -1$, we pick out the transitions indicated as I in figure 2. From (2.20a), the corresponding modes have $\boldsymbol{q}_{s\perp} = -2K_m\hat{\boldsymbol{y}}$. If $w = +1$, then we pick out the transitions indicated as II in figure 2 and the corresponding modes have $\boldsymbol{q}_{s\perp} = 2K_m\hat{\boldsymbol{y}}$. Let us define

$$\Delta_n = [\hbar^2\omega_s^2 - 4\hbar^2c^2K_m^2\epsilon_\infty^{-1}]^{1/2}. \tag{2.25}$$

Since the amplitudes of the coherently generated modes are proportional to

$$|\hat{\boldsymbol{z}} \cdot \boldsymbol{q}_s|^{-1} = \hbar c\epsilon_\infty^{-1/2}\Delta_n^{-1} \tag{2.26}$$

the scattering is resonantly enhanced if the magnitude of the magnetic field is adjusted so that Δ_n is extremely small. If the Landau levels are sufficiently well separated, this enhancement will pick out a definite value of n.

Note that the above adjustment can readily be made because the wavevector K_m can be large for relatively weak magnetic fields, yielding an energy which is comparable to typical band gaps. For example, if $B_0 = 10$ kG, $\epsilon_\infty = 16$, and $z_0 = L = 60$ Å, then

$$2\hbar cK_m\epsilon_\infty^{-1/2} = eB_0(2z_0 + L)\epsilon_\infty^{-1/2} \simeq 1.5 \text{ eV}. \tag{2.27}$$

Equation (2.25) suggests that Δ_n can go to zero, causing an enhancement of infinite magnitude. In fact this will not happen, since electronic states have finite lifetimes and δ in equation (2.21) should be replaced by an appropriate energy width γ. This will replace Δ_n by

$$\bar{\Delta}_n = [\Delta_n^2 + \gamma^2]^{1/2}. \tag{2.28}$$

Finite times for electronic states does not contradict the ballistic motion hypothesis per se, since the latter requires simply that $\mathcal{L}_y \lesssim l_s$, where l_s is the mean free path for electrons. It should also be noted that there are actually two distinct decay times. One is concerned with the off-diagonal density matrix elements like ρ^{AB}. The other is concerned with the diagonal density matrix elements. γ should be related to the latter.

Going back to the mode equations (2.20a), we have four propagation vectors:

$$\boldsymbol{q}_s = \pm\hat{\boldsymbol{y}}\,2K_m \pm \hat{\boldsymbol{z}}\epsilon_\infty^{1/2}(\hbar c)^{-1}\bar{\Delta}_n. \tag{2.29}$$

As $\bar{\Delta}_n \to \gamma$, the z-component of $\boldsymbol{q}_s$ is nearly zero and the two modes begin to merge. Polarization vectors may be obtained from (2.22). Let us consider only the mode with the plus z-component and the minus y-component (designating it by $s = -+$) and let its polarization vector be

$$\hat{\epsilon}_{-+} = -(\hat{\boldsymbol{y}}\cos\theta_n + \hat{\boldsymbol{z}}\sin\theta_n) \qquad \theta_n = \tan^{-1}\left(\frac{2\hbar c K_m}{\sqrt{\epsilon_\infty}\bar{\Delta}_n}\right). \tag{2.30}$$

Again as $\bar{\Delta}_n \to \gamma$, $\hat{\epsilon}_{-+}$ is nearly $-\hat{\boldsymbol{z}}$. Consider the matrix elements M in (2.21). For $w'' = +1$, we have

$$\begin{aligned} M^*_{cn-;vn''+}(0,0|\hat{\boldsymbol{z}}\cdot\boldsymbol{q}_s)M_{cn+;vn''+}(0,0|q_i) &\simeq M^*_{cn-;vn''+}(0,0|0)M_{cn+;vn''+}(0,0|0) \\ &= \exp(-l_B^2K_m^2)D_{nn''}(-2l_BK_m)D_{nn''}(0) \\ &= \delta_{nn''}\exp)(-l_B^2K_m^2)D_{nn}(2l_BK_m). \end{aligned} \tag{2.31}$$

The last two lines follow from (2.10). Thus within the validity of the approximations here, one must set $n = n''$ in (2.24). For $w'' = -1$, we have, using the symmetry properties of ϕ_{bnwK_y} given in appendix A (equations (A7a) and (A7b))

$$\begin{aligned} &M^*_{cn-;vn''-}(0,-2K_m|\hat{\boldsymbol{z}}\cdot\boldsymbol{q}_s)M_{cn+;vn''-}(0,-2K_m|q_i) \\ &\quad = M^*_{cn-;vn''+}(0,0|\hat{\boldsymbol{z}}\cdot\boldsymbol{q}_s)M_{cn+;vn''+}(0,0|q_i). \end{aligned} \tag{2.32}$$

Therefore $w'' = \pm1$ produce identical terms in (2.21). Using (2.31) and (2.32) in (2.21), one finds that for the $s = -+$ mode

$$\begin{aligned} (\boldsymbol{A}(\boldsymbol{x},t))^{(2)}_{-+} \simeq \hat{\epsilon}_{-+}&\left(\frac{2\pi\hbar\bar{n}_i}{\epsilon_\infty V_{ol}\omega_i}\right)^{1/2}\left(\frac{2\pi e^2N\hbar}{m^2\sqrt{\epsilon_\infty}\bar{\Delta}_n}\right)\exp(-l_B^2K_m^2) \\ &\times D_{nn}(2l_BK_m)\mathrm{Re}\left\{\frac{\mathrm{i}\bar{\rho}^{\mathrm{AB}}_{cn-;cn+}\hat{\epsilon}_{-+}\cdot\boldsymbol{P}^*_{cv}\hat{\epsilon}_i\cdot\boldsymbol{P}_{cv}}{(\hbar\omega_{-+}-\hbar\omega_i+\mathrm{i}\gamma)}\right. \\ &\left.\times\exp[\mathrm{i}(\boldsymbol{q}_{-+}\cdot\boldsymbol{x}-\omega_{-+}t+\phi_i)]\right\}. \end{aligned} \tag{2.33}$$

For the purpose of comparison, it is convenient to take the absolute value of the ratio of the complex amplitude of the scattered wave to the complex amplitude of the incident wave. Defining this ratio as r_{-+}, one finds

$$r_{-+} = \frac{\pi e^2N\hbar\exp(-l_B^2K_m^2)}{m^2c\sqrt{\epsilon_\infty}\bar{\Delta}_n[(\hbar\omega_{-+}-\hbar\omega_i)^2+\gamma^2]^{1/2}}\left|\bar{\rho}^{\mathrm{AB}}_{cn-;cn+}\hat{\epsilon}_{-+}\cdot\boldsymbol{P}^*_{cv}\hat{\epsilon}_i\cdot\boldsymbol{P}_{cv}D_{nn}(2l_BK_m)\right|. \tag{2.34}$$

r_{-+}^2 compares the densities of the quanta in the two waves and therefore yields the efficiency of the scattering mechanism into this s-mode.

In order to have a finite r_{-+}, the orientation of the structure must be such that the products $\hat{\epsilon}_{-+} \cdot \boldsymbol{P}_{cv}^* \hat{\epsilon}_i \cdot \boldsymbol{P}_{cv}$ do not vanish. In the geometry of figure 1, if we assume that $\vec{P}_{cv}$ is in the [111] direction,

$$\left|\hat{\epsilon}_{-+} \cdot \boldsymbol{P}_{cv}^* \hat{\epsilon}_i \cdot \boldsymbol{P}_{cv}\right|^2 \simeq \tfrac{1}{3}\left|\boldsymbol{P}_{cv}\right|^2. \qquad (2.35)$$

Typically $|\boldsymbol{P}_{cv}|$ corresponds to a large wavevector for the III–V semiconductor compounds: $|\boldsymbol{P}_{cv}|/\hbar \sim 10^8$ cm^{-1}. As before let us assume that $z_0 = L = 60$ Å, $B_0 = 10$ kG, and $\epsilon_\infty = 16$. For $m_c^* = m_v^* = 0.1$ m, there is a single cyclotron frequency: $\hbar\omega_c \simeq 10^{-3}$ eV for the above field strength. Assume that the energy width γ is comparable to the one tenth of the cyclotron energy: $\gamma \sim 10^{-4}$ eV. The peak values of r_{-+} are obtained when the resonance conditions

$$\hbar\omega_i \simeq E_G + \hbar\omega_c(2n+1) + \delta W_{cn} + \delta W_{vn} \simeq eB_0\epsilon_\infty^{-1/2}(2z_0 + L) \qquad (2.36)$$

are met. Let the injected electrons be in the $n = 0$ level for the sake of simplicity. This yields $D_{00} = 1$. Since the double-quantum-well structure is symmetric, it is reasonable to set $|\bar{\rho}^{AB}_{c0-;c0+}| = 1/\sqrt{2}$. For ordinary electrons, typical injection densities into quantum wells can be on the order of 10^{12} cm^{-2}. Let us assume that ballistic electrons which are in Aharonov–Bohm superposition states form an extremely small fraction of such densities, say $N \sim 5 \times 10^8$ cm^{-2}. Under the resonance conditions (2.36), one then finds that $r_{-+} \simeq 0.1$. This is a substantial value for r_{-+} and indicates that the proposed coherent scattering mechanism can be extremely efficient.

The intermediate band b'' in (2.21) may be a higher conduction band instead of a valence band. The coherent scattering is then into the s'-modes in (2.20*b*) and (2.21). In this case the order of absorptions and emissions are interchanged compared with the previous case. For example, the state $|cn-\rangle$ transits into $|c'n''-\rangle$ by the absorption of an ω_i-photon, where c' designates the higher conduction band. $|c'n''-\rangle$ transits into $|cn+\rangle$ by the emission of an $\omega_{s'}$-photon. $|cn+\rangle$ undergoes a similar sequence of transitions. If we let $\sigma_{c'} = \sigma_c = +1$, the frequency of the coherently scattered modes is given by

$$\hbar\omega_{s'} = E_{c'0} - E_{c0} + \hbar(\omega_{c'c} - \omega_{cc})(n + \tfrac{1}{2}) + \delta W_{c'n} - \delta W_{cn} \qquad (2.37)$$

where $E_{c'0}$, E_{c0} designate the band edges. When ω_s is replaced by $\omega_{s'}$ in equations (2.25) and (2.29), these equations also hold for the s'-modes. Furthermore, the additional replacement of $\boldsymbol{P}_{cv}$ by $\boldsymbol{P}_{c'c}$ in (2.34) yields $r_{s'}$. The resonance conditions for the s'-modes become

$$\hbar\omega_i \simeq E_{c'0} - E_{c0} + \hbar(\omega_{c'c} - \omega_{cc})(n + \tfrac{1}{2}) + \delta W_{c'n} - \delta W_{cn} \simeq eB_0\epsilon_\infty^{-1/2}(2z_0 + L). \qquad (2.38)$$

The factor ρ^{AB} in the preceding field equations of the scattering modes indicates *that the proposed scattering mechanism can be realized only through electrons which* are in Aharonov–Bohm superposition states. If one were to consider thermal electrons or electrons whose orbits are centred just in one well with a density matrix operator of the form $\rho^{e}_{bnw;b'n'w'}$, one would find that radiative transitions are associated with electronic transitions within a given well. There would be no effective charge circulation around the magnetic field lines. The scattering described by (2.21) would reduce to the usual light scattering involving interfaces. Namely, one part of the incident light field would propagate forward in the original mode, while another part would be reflected back. Therefore one can use the new coherent scattering mechanism as a test of the existence and number of ballistic electrons which are in Aharonov–Bohm superposition states. Such optical measurements would be more discriminating than those involving conductivity.

In the discussion above, we ignored the effects of the DC field $\mathcal{E}_y$. For sufficiently large $\mathcal{E}_y$ (but still within the approximation that Eq. (2.4) holds), the matrix element M defined in (2.9b) has terms which are linearly dependent on $\mathcal{E}_y$. These are the correction terms to (2.10a) and can be written as

$$M^{(1)}_{bnw;b'n'w'}(K_y, K'_y|q) = \mathrm{i}\int \mathrm{d}z\ \exp(\mathrm{i}qz) \times [\sigma_b\kappa_b(z - wl_B^2K_m - l_B^2K_y) - \sigma_{b'}\kappa_{b'}(z - w'l_B^2K_m - l_B^2K'_y)] \times u_n(z - wl_B^2K_m - l_B^2K_y)u_{n'}(z - w'l_B^2K_m - l_B^2K'_y). \tag{2.39}$$

As K_y, K'_y, $q_z \to 0$, $M^{(1)}$ can produce transitions which obey the selection rule $n = n' \pm 1$ in addition to those obeying the main one, $n = n'$. This comes about from the fact that the integrand in (2.39) is proportional to $zu_nu_{n'}$.

We should note that a similar violation of the rule $n = n'$ can arise from the $\beta^b_{nn'}$-corrections to the Landau wavefunction for sufficiently large quantum well potentials. Both the selection rules and the resonance conditions are modified if the field $\mathcal{E}_z$ is also allowed to be large.

3. Concluding remarks

In the preceding section we discussed the light scattering for a structure in which the wells are of identical width. This symmetry simplifies the algebra and the results. However, the proposed light scattering does not depend on this symmetry and the formulas may easily be adapted to the case when the structure is not symmetric by modifying the reduction of the electronic wavevector defined in (1.2a). When the widths of the wells differ, one can define

$$\boldsymbol{k} = \boldsymbol{K} + \hat{y}K_m^{(w)} \tag{3.1a}$$

$$K_m^{(\pm)} = \pm\frac{1}{l_B^2}\left(z_0 + \tfrac{1}{2}L_\pm\right) \tag{3.1b}$$

where $L_\pm$ are the widths of the wells on the right and on the left of the origin, respectively. Thus $w = \pm 1$ may still be treated as a quantum number for the states whose orbital centres lie in the wells. Substitution of wK_m by $K_m^{(w)}$ in (2.18a)–(2.18d), (2.19), (2.20a) and (2.20b), and (2.21) yields the formulas for a non-symmetric double-well structure.

The typical parametric values cited following (2.35) indicate that the new coherent light scattering should be relatively easily observed experimentally in a double-quantum-well structure similar to the one used by Datta *et al* (1985), but with narrower wells and a closer distance between them. One can construct a series of such microstructures (identical in length and width) in order to increase the utilization of the incident light. The coherently generated modes can come out in directions which are nearly in the plane of the quantum wells; thus, they are easily distinguishable from the incident field as well as from other scattering modes which may be present. By varying the magnetic field strength, one can resonantly enhance certain mode frequencies, as well as alter propagation directions. With suitably chosen high mobility materials, one may improve on the values for N and γ given above. The new coherent scattering mechanism can be quite efficient. We should note, however, that our calculation in section 2 and in appendix B is a perturbative one, predicated on the assumption that scattering amplitudes will be small compared with incident field amplitudes. We expect this calculation to be invalid as r_s increases to values which are near 1. In that regime one should perform a non-perturbative, self-consistent calculation.

The analysis of this paper is based on single particle excitations. A more rigorous analysis would need to treat the problem in a many-body framework, specifically taking into account electron-electron scatterings, which might have an effect on ρ^{AB}. However, the ρ^{AB} that appears in the coherent light scattering calculations above also appears in the part of the conductivity which displays the Aharonov–Bohm interferences. Since we have the empirical evidence from the experiment of Datta *et al* that these interferences take place, electron–electron scatterings are not capable of reducing ρ^{AB} to zero. Any sample that exhibits the Aharonov–Bohm effect in its conductivity will also exhibit the proposed magnetic-flux-induced coherent light scattering.

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Appendix A

To evaluate the coupling coefficient g from ψ_{bnwK}, we note that the functions multiplying ψ^B_{b0} are slowly varying compared with ψ^B_{b0}. This fact may be used to obtain a simple expression for g. Consider the matrix element $\langle\alpha|\exp(\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x})\boldsymbol{p}|\alpha'\rangle$, where $\alpha=(bnw\boldsymbol{K})$, $\alpha'=(b'n'w'\boldsymbol{K}')$, and $b\neq b'$:

$$\langle\alpha|\exp(\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x})\boldsymbol{p}|\alpha'\rangle=\int\frac{\mathrm{d}\boldsymbol{x}}{\mathcal{L}_x\mathcal{L}_y}\exp(\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x})\exp[-\mathrm{i}(\boldsymbol{K}+wK_m\hat{\boldsymbol{y}})\cdot\boldsymbol{x}]$$
$$\times\,\phi^*_{bnwK_y}\psi^{B*}_{b0}\left(-\mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{x}}\right)\exp[\mathrm{i}(\boldsymbol{K}'+w'K_m\hat{\boldsymbol{y}})\cdot\boldsymbol{x}]\phi_{b'n'w'K'_y}\psi^B_{b'0}.\qquad\text{(A1)}$$

Let us set $\boldsymbol{x}=\boldsymbol{\xi}+\boldsymbol{R}$, where $\boldsymbol{\xi}$ varies within a unit cell of volume Ω and $\boldsymbol{R}$ designates the unit cell position. The integral over the crystal may then be converted into an integral over a reference cell and a sum over the unit cells:

$$\int\mathrm{d}\boldsymbol{x}=\sum_{\boldsymbol{R}}\int_\Omega\mathrm{d}\boldsymbol{\xi}.\qquad\text{(A2)}$$

ψ_{b0}^B and $\psi_{b'0}^B$ are periodic in $\boldsymbol{R}$. Since the functions multiplying the Bloch functions are slowly varying, we let them to be functions of just $\boldsymbol{R}$. Equation (A1) becomes

$$\langle\alpha|\exp(\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x})\boldsymbol{p}|\alpha'\rangle \simeq \frac{1}{\mathcal{L}_x\mathcal{L}_y}\sum_{\boldsymbol{R}}\exp[-\mathrm{i}(\boldsymbol{K}-\boldsymbol{K}'-\boldsymbol{q}_\perp+wK_m\hat{\boldsymbol{y}}-w'K_m\hat{\boldsymbol{y}})\cdot\boldsymbol{R}]$$
$$\times\left[\exp(\mathrm{i}q_zR_z)\phi^*_{bnwK_y}(R_z)\phi_{b'n'w'K'_y}(R_z)\right]$$
$$\times\int_\Omega \mathrm{d}\boldsymbol{\xi}\,\psi_{b0}^{B*}(\boldsymbol{\xi})\left(-\mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{\xi}}\right)\psi_{b'0}^B(\boldsymbol{\xi}). \tag{A3}$$

The integral over the unit cell gives the interband matrix element of the momentum operator:

$$\boldsymbol{P}_{bb'} = \frac{1}{\Omega}\int_\Omega \mathrm{d}\boldsymbol{\xi}\,\psi_{b0}^{B*}(\boldsymbol{\xi})\left(-\mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{\xi}}\right)\psi_{b'0}^B(\boldsymbol{\xi}). \tag{A4}$$

We substitute (A4) into (A3) and convert the sum back to an integral:

$$\frac{\Omega}{\mathcal{L}_x\mathcal{L}_y}\sum_{\boldsymbol{R}} = \int\frac{\mathrm{d}\boldsymbol{R}}{\mathcal{L}_x\mathcal{L}_y} = \int\frac{\mathrm{d}\boldsymbol{R}_\perp}{\mathcal{L}_x\mathcal{L}_y}\int \mathrm{d}R_z. \tag{A5}$$

The matrix element becomes

$$\langle\alpha|\exp(\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x})\boldsymbol{p}|\alpha'\rangle \simeq \delta_{\boldsymbol{K}+wK_m\hat{\boldsymbol{y}};\boldsymbol{K}'+w'K_m\hat{\boldsymbol{y}}+\boldsymbol{q}_\perp}\boldsymbol{P}_{bb'}M_{bnw;b'n'w'}(K_y,K'_y|q_z) \tag{A6}$$

where $M_{bnw;b'n'w'}$ is given by (2.9*b*). When (A6) is multiplied with the appropriate coefficients coming from the vector field, one obtains (2.9*a*).

In the evaluation of the transition matrix elements in Eq. (2.32), we exploited the symmetry of the structure in Figs. 1 and 2. If the wells are identical, ϕ_{bnwK_y} has the symmetry

$$\phi_{bn,w=+1,K_y=-2K_m} = \phi_{bn,w=-1,K_y=0} \tag{A7a}$$
$$\phi_{bn,w=-1,K_y=2K_m} = \phi_{bn,w=+1,K_y=0}. \tag{A7b}$$

Appendix B

Let $H_I(t)$ represent H_1 in the interaction representation:

$$H_I(t) = \exp(\mathrm{i}H_0t/\hbar)H_1\exp(-\mathrm{i}H_0t)$$
$$= \sum_{\alpha\alpha'\mu}\left[\hbar g^\mu_{\alpha\alpha'}\exp[\mathrm{i}(E_\alpha-E_{\alpha'}-\hbar w_\mu)t/\hbar]|\alpha\rangle\langle\alpha'|b_\mu\right.$$
$$\left.+\hbar g^{\mu^*}_{\alpha\alpha'}\exp[-\mathrm{i}(E_\alpha-E'_\alpha-\hbar\omega_\mu)t/\hbar]|\alpha'\rangle\langle\alpha|b^\dagger_\mu\right] \tag{B1}$$

The transverse vector potential field operator in the interaction representation is given by

$$A_I(x,t) = \sum_{\mu} \hat{\epsilon}_\mu \left[A_\mu(x)\exp(-\mathrm{i}\omega_\mu t)b_\mu + A^*_\mu(x)\exp(\mathrm{i}\omega_\mu t)b^\dagger_\mu\right] \tag{B2a}$$

where

$$A_\mu(x) = \left(\frac{2\pi\hbar c^2}{\omega_\mu V_{ol}\epsilon_\infty}\right)^{1/2} \exp(\mathrm{i}q_\mu \cdot x). \tag{B2b}$$

In the interaction representation,the density matrix operator for one electron plus the radiation fields is given, to second order in $H_I(t)$, by

$$\rho_I(t) \simeq \rho(0) + [R(t),\rho(0)] + \tfrac{1}{2}[R(t),[R(t),\rho(0)]] \tag{B3}$$

where

$$\begin{aligned} R(t) &= -\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}t'\, H_I(t') \\ &= \sum_{\alpha\alpha'\mu}\left[-g^{\mu}_{\alpha\alpha'}\zeta\left(\frac{E_\alpha - E_{\alpha'}}{\hbar} - \omega_\mu; t\right)|\alpha\rangle\langle\alpha'|b_\mu \right. \\ &\quad \left. + g^{\mu^*}_{\alpha\alpha'}\zeta^*\left(\frac{E_\alpha - E_{\alpha'}}{\hbar} - \omega_\mu; t\right)|\alpha'\rangle\langle\alpha|b^\dagger_\mu\right] \\ &= \sum_{\alpha\alpha'\mu}[-R^{\mu}_{\alpha\alpha'}(t)|\alpha\rangle\langle\alpha'|b_\mu + R^{\mu^*}_{\alpha\alpha'}(t)|\alpha'\rangle\langle\alpha|b^\dagger_\mu] \end{aligned} \tag{B4a}$$

and

$$\zeta(x;t) = [\exp(\mathrm{i}xt) - 1]/x. \tag{B4b}$$

From (B3) one finds that, for $\mathcal{N}$ electrons, the expectation value of the transverse vector potential field operator in the second order is given by

$$\langle A(x,t)\rangle^{(2)} = \tfrac{1}{2}\mathcal{N}\mathrm{Tr}\,([[A_I(x,t),R(t)],R(t)]\rho(0)). \tag{B5}$$

From (2.14)–(2.16) and (B3), one finds after some algebra that

$$\begin{aligned} \langle A(x,t)\rangle^{(2)} = \mathcal{N}\sum_{\mu\nu\alpha\beta\gamma}\hat{\epsilon}_\mu \mathrm{Re}\left\{A_\mu(x)\exp(-\mathrm{i}\omega_\mu t)\rho^{e}_{\alpha\beta}\left[R^{\mu^*}_{\alpha\gamma}R^{\nu}_{\beta\gamma}\langle b_\nu\rangle - R^{\mu^*}_{\alpha\gamma}R^{\nu*}_{\gamma\beta}\langle b^\dagger_\nu\rangle \right.\right. \\ \left.\left. - R^{\mu^*}_{\gamma\beta}R^{\nu}_{\gamma\alpha}\langle b_\nu\rangle + R^{\nu^*}_{\alpha\gamma}R^{\mu^*}_{\gamma\beta}\langle b^\dagger_\nu\rangle\right]\right\}. \end{aligned} \tag{B6}$$

As t becomes indefinitely large, the function ζ can be written in the form:

$$\zeta\left(\frac{E_\alpha - E_{\alpha'}}{\hbar} - \omega_\mu; t\right) \longrightarrow \lim_{\delta\to 0}\left(\frac{-\hbar}{E_\alpha - E_{\alpha'} - \hbar\omega_\mu + \mathrm{i}\delta}\right). \tag{B7}$$

Thus

$$R^{\mu}_{\alpha\alpha'} \longrightarrow \lim_{\delta\to 0}\left(\frac{-\hbar g^{\mu}_{\alpha\alpha'}}{E_\alpha - E_{\alpha'} - \hbar\omega_\mu + \mathrm{i}\delta}\right)$$
$$= -\Pr\left(\frac{\hbar g^{\mu}_{\alpha\alpha'}}{E_\alpha - E_{\alpha'} - \hbar\omega_\mu}\right) + \mathrm{i}\pi\hbar g^{\mu}_{\alpha\alpha'}\delta(E_\alpha - E_{\alpha'} - \hbar\omega_\mu). \tag{B8}$$

The coherently scattered modes come from the delta function:

$$\langle \boldsymbol{A}(\boldsymbol{x},t)\rangle^{(2)}_{\mathrm{coh}} = \mathcal{N}\sum_{\mu\alpha\beta\gamma}\hat{\epsilon}_\mu \mathrm{Re}\left\{\mathrm{i}\pi\hbar A_\mu(\boldsymbol{x})\exp(-\mathrm{i}\omega_\mu t)\rho^{\mathrm{e}}_{\alpha\beta}\right.$$
$$\times\left[g^{\mu^*}_{\alpha\gamma}\delta(E_\alpha - E_\gamma - \hbar\omega_\mu)(R^{i^*}_{\gamma\beta}\langle b_i\rangle^* - R^{i}_{\beta\gamma}\langle b_i\rangle)\right.$$
$$\left.\left. + g^{\mu^*}_{\gamma\beta}\delta(E_\gamma - E_\beta - \hbar\omega_\mu)(R^{i}_{\gamma\alpha}\langle b_i\rangle - R^{i^*}_{\alpha\gamma}\langle b_i\rangle^*)\right]\right\}. \tag{B9}$$

Here we used the fact that the incident light is a single coherent mode. When (B9) is specialized to electrons in Aharonov–Bohm superpositions by using (2.17), one obtains

$$\langle \boldsymbol{A}(\boldsymbol{x},t)\rangle^{(2)}_{\mathrm{coh}} = \pi\mathcal{N}\hbar^2 c\left(\frac{2\pi\hbar\bar{n}_i}{\epsilon_\infty V_{ol}}\right)^{1/2}$$
$$\times\sum_{\mu b n n' w \boldsymbol{K}\boldsymbol{K}'}\frac{\hat{\epsilon}_\mu}{\sqrt{\omega_\mu}}\mathrm{Re}\left\{\mathrm{i}\exp[\mathrm{i}(\boldsymbol{q}_\mu\cdot\boldsymbol{x} - \omega_\mu t)]\rho^{\mathrm{AB}}_{bnw;bn'-w}(\boldsymbol{K},\boldsymbol{K}')\right.$$
$$\sum_{b''n''\boldsymbol{K}''w''}\left[\frac{\exp(\mathrm{i}\phi_i)}{(\hbar\omega_\mu - \hbar\omega_i + \mathrm{i}\delta)}g^{\mu*}_{bnw;b''n''w''}(\boldsymbol{K},\boldsymbol{K}'')g^{i}_{bn'-w;b''n''w''}\right.$$
$$\times(\boldsymbol{K}',\boldsymbol{K}'')\delta\left[E_{bnw}(\boldsymbol{K}) - E_{b''n''w''}(\boldsymbol{K}'') - \hbar\omega_\mu\right]$$
$$+\frac{\exp(-\mathrm{i}\phi_i)}{(\hbar\omega_\mu + \hbar\omega_i + \mathrm{i}\delta)}g^{\mu*}_{bnw;b''n''w''}(\boldsymbol{K},\boldsymbol{K}'')g^{i*}_{b''n''w'';bn'-w}(\boldsymbol{K}'',\boldsymbol{K}')$$
$$\times\delta\left[E_{bnw}(\boldsymbol{K}) - E_{b''n''w''}(\boldsymbol{K}'') - \hbar\omega_\mu\right]$$
$$-\frac{\exp(\mathrm{i}\phi_i)}{(\hbar\omega_\mu - \hbar\omega_i + \mathrm{i}\delta)}g^{\mu*}_{b''n''w'';bn'-w}(\boldsymbol{K}'',\boldsymbol{K}')g^{i}_{b''n''w'';bnw}(\boldsymbol{K}'',\boldsymbol{K})$$
$$\times\delta\left[E_{b''n''w''}(\boldsymbol{K}'') - E_{bn'-w}(\boldsymbol{K}') - \hbar\omega_\mu\right]$$
$$-\frac{\exp(-\mathrm{i}\phi_i)}{(\hbar\omega_\mu + \hbar\omega_i + \mathrm{i}\delta)}g^{\mu*}_{b''n''w'';bn'-w}(\boldsymbol{K}'',\boldsymbol{K}')g^{i*}_{bnw;b''n''w''}(\boldsymbol{K},\boldsymbol{K}')$$
$$\left.\left.\times\delta\left[E_{b''n''w''}(\boldsymbol{K}'') - E_{bn'-w}(\boldsymbol{K}') - \hbar\omega_\mu\right]\right]\right\}. \tag{B10}$$

Although the states $|bnw\boldsymbol{K}\rangle$ and $|bn'-w\boldsymbol{K}'\rangle$ are centred in the wells, we do not impose *a priori* such a condition on the intermediate states in (B10). $\boldsymbol{K}''$ may take on arbitrary values in the sum in (B10). However, when (B10) is evaluated, the coherent generation of scattered fields forces the intermediate states to be centred in the wells.

Let us designate the terms of (B10) in the square brackets, together with the sums over $b''n''w''\boldsymbol{K}''$, as

$$\left[\exp(\mathrm{i}\phi_i)T_1 + \exp(-\mathrm{i}\phi_i)T_2 - \exp(\mathrm{i}\phi_i)T_3 - \exp(-\mathrm{i}\phi_i)T_4\right].$$

From (2.9*a*) and (2.17) one finds

$$T_1 = \frac{2\pi e^2 \delta_{\boldsymbol{K}-\boldsymbol{K}'+2wK_m\hat{\boldsymbol{y}};\boldsymbol{q}_{\mu\perp}-\boldsymbol{q}_{i\perp}}}{\hbar m^2 \epsilon_\infty V_{ol}\sqrt{\omega_\mu \omega_i}(\hbar\omega_\mu - \hbar\omega_i + \mathrm{i}\delta)} \sum_{b''n''w''} \hat{\epsilon}_\mu \cdot \boldsymbol{P}^*_{bb''}\hat{\epsilon}_i \cdot \boldsymbol{P}_{bb''}$$
$$\times M^*_{bnw;b''n''w''}(K_y, K'_y - (w+w'')K_m - q_{iy}|q_{\mu z})$$
$$\times M_{bn'-w;b''n''w''}(K'_y, K'_y - (w+w'')K_m - q_{iy}|q_{iz})$$
$$\times \delta\left[E_{bn'-w}(\boldsymbol{K}') - E_{b''n''w''}(\boldsymbol{K}' - (w+w'')K_m\hat{\boldsymbol{y}} - \boldsymbol{q}_{i\perp}) - \hbar\omega_\mu\right] \quad \text{(B11}a\text{)}$$

$$T_2 = \frac{2\pi e^2 \delta_{\boldsymbol{K}-\boldsymbol{K}'+2wK_m\hat{\boldsymbol{y}};\boldsymbol{q}_{\mu\perp}+\boldsymbol{q}_{i\perp}}}{\hbar m^2 \epsilon_\infty V_{ol}\sqrt{\omega_\mu \omega_i}(\hbar\omega_\mu + \hbar\omega_i + \mathrm{i}\delta)} \sum_{b''n''w''} \hat{\epsilon}_\mu \cdot \boldsymbol{P}^*_{bb''}\hat{\epsilon}_i \cdot \boldsymbol{P}^*_{b''b}$$
$$\times M^*_{bnw;b''n''w''}(K_y, K'_y - (w+w'')K_m + q_{iy}|q_{\mu z})$$
$$\times M^*_{b''n''w'';bn'-w}(K'_y - (w+w'')K_m + q_{iy}, K'_y|q_{iz})$$
$$\times \delta\left[E_{bn'-w}(\boldsymbol{K}') - E_{b''n''w''}(\boldsymbol{K}' - (w+w'')K_m\hat{\boldsymbol{y}} + \boldsymbol{q}_{i\perp}) - \hbar\omega_\mu\right] \quad \text{(B11}b\text{)}$$

$$T_3 = \frac{2\pi e^2 \delta_{\boldsymbol{K}-\boldsymbol{K}'+2wK_m\hat{\boldsymbol{y}};\boldsymbol{q}_{\mu\perp}-\boldsymbol{q}_{i\perp}}}{\hbar m^2 \epsilon_\infty V_{ol}\sqrt{\omega_\mu \omega_i}(\hbar\omega_\mu - \hbar\omega_i + \mathrm{i}\delta)} \sum_{b''n''w''} \hat{\epsilon}_\mu \cdot \boldsymbol{P}^*_{b''b}\hat{\epsilon}_i \cdot \boldsymbol{P}_{b''b}$$
$$\times M^*_{b''n''w'';bn'-w}(K_y + (w-w'')K_m + q_{iy}, K'_y|q_{\mu z})$$
$$\times M_{b''n''w'';bnw}(K_y + (w-w'')K_m + q_{iy}, K_y|q_{iz})$$
$$\times \delta\left[E_{b''n''w''}(\boldsymbol{K} + (w-w'')K_m\hat{\boldsymbol{y}} + \boldsymbol{q}_{i\perp}) - E_{bnw}(\boldsymbol{K}) - \hbar\omega_\mu\right] \quad \text{(B11}c\text{)}$$

$$T_4 = \frac{2\pi e^2 \delta_{\boldsymbol{K}-\boldsymbol{K}'+2wK_m\hat{\boldsymbol{y}};\boldsymbol{q}_{\mu\perp}+\boldsymbol{q}_{i\perp}}}{\hbar m^2 \epsilon_\infty V_{ol}\sqrt{\omega_\mu \omega_i}(\hbar\omega_\mu + \hbar\omega_i + \mathrm{i}\delta)} \sum_{b''n''w''} \hat{\epsilon}_\mu \cdot \boldsymbol{P}^*_{b''b}\hat{\epsilon}_i \cdot \boldsymbol{P}^*_{bb''}$$
$$\times M^*_{b''n''w'';bn'-w}(K_y + (w-w'')K_m - q_{iy}, K'_y|q_{\mu z})$$
$$\times M^*_{bnw;b''n''w''}(K_y, K_y + (w-w'')K_m - q_{iy}|q_{iz})$$
$$\times \delta\left[E_{b''n''w''}(\boldsymbol{K} + (w-w'')K_m\hat{\boldsymbol{y}} - \boldsymbol{q}_{i\perp}) - E_{bnw}(\boldsymbol{K}) - \hbar\omega_\mu\right]. \quad \text{(B11}d\text{)}$$

Next we perform the integration over the radiation modes in (B10). For the T_1-term this integration has the form

$$\sum_\mu \frac{\hat{\epsilon}_\mu}{\sqrt{\omega_\mu}} \exp[\mathrm{i}(\boldsymbol{q}_\mu \cdot \boldsymbol{x} - \omega_\mu t)]T_1 = \sum_{q_{\mu 1} q_{\mu 2}} \left(\frac{\mathcal{L}_z}{2\pi}\right) \int \mathrm{d}q_{\mu 3}\, \frac{\hat{\epsilon}_\mu}{\sqrt{\omega_\mu}} \exp[\mathrm{i}(\boldsymbol{q}_\mu \cdot \boldsymbol{x} - \omega_\mu t)]T_1 \quad \text{(B12)}$$

The same form is obtained for T_2, T_3 and T_4 terms. There are three delta functions in the integrand of (B12). Because of these delta functions, the integral is trivial and reduces to a sum over the modes which satisfy the delta functions. For the T_1 integral, these modes represent the coherently generated modes and are the solutions of (2.18*a*). For T_2, T_3, and T_4 integrations, the coherently generated modes are the solutions of (2.18*b*), (2.18*c*), and (2.18*d*), respectively. Using (B11*a*)–(B11*d*) and (2.18*a*)–(2.18*d*) in (B10), one obtains the expression in (2.19).

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